TECHNICAL STABILITY OF PANEL MOTION IN A GAS STREAM

K. S. Matviichuk

UDC 531.36:534.1

The vibrations problem of panels representing extensively utilized structural elements is of practical importance in mechanics and engineering. As is known, such panel vibrations developing in the supersonic domain may cause instability of the structure. Among the reasons specifying the panel vibrations should be primarily aerodynamic forces that occur due to motion. The papers [1-16] are devoted to the panel flutter problem, i.e., the interaction of aerodynamic forces and panel motion. Since this problem is so very complex, simplifying assumptions are ordinarily made in many investigations and the stability properties are studied in panel flutter problems on the basis of applying different direct integration methods. One of the effective methods of an analytic study of stability in panel flutter problems is the direct Lyapunov method [8-17].

Sufficient conditions are obtained in this paper for the technical stability (TS) [17-24] of two-dimensional panel motion in a supersonic stream. The fundamental results are found on the basis of the comparison method [22-24] in combination with the second Lyapunov method [17, 21]. According to [8-14], the motion of the system under consideration is described by a linear partial differential equation obtained on the basis of piston theory. A load applied to a panel along its edge is independent of the time. The TS domain is related to positive definiteness condition of the Lyapunov functional, a small positive parameter, and regularity conditions for the solution of the appropriate Cauchy scalar comparison problem by means of the Mach number. The TS of the process in finite and infinite time intervals and the asymptotic TS are investigated.

Sufficient Lyapunov stability conditions are found in [9-16] for an analogous system. The results of this paper differ substantially from the stability properties in the sense of Lyapunov mentioned in [9-16] not only by the fact that the TS conditions of the system under consideration are studied in any finite and previously assigned time interval but also by the fact that constraints on the initial state of the system are independent of the majorization conditions of subsequent states of the process during the given time interval. Upon satisfying specific conditions the domain found here for the Mach number values includes analogous domains in [12-16]. Conditions are mentioned for which technical instability is possible for the original process. The approach proposed in this paper, based on the comparison method in combination with the second Lyapunov method can be applied to investigate TS properties in more complex panel flutter problems without simplifying assumptions, for instance, in nonlinear panel flutter problems, in the same problems in the presence of parametric loadings in the case of curved panels with different boundary clamping methods, cylindrical, conical, or tuncated conical panels, as well as in problems of panel vibrations due to either aerodynamic noise or buffeting. The absence of negative-definiteness conditions on the total derivative of the Lyapunov functional by virtue of the original boundary value problem here expands the possibility for conditions on the dynamic process parameters in this approach, in contrast to the stability in the Lyapunov sense.

1. FORMULATION OF THE PROBLEM AND CONDITIONS ON THE LYAPUNOV FUNCTIONAL

Let us consider the dynamic behavior of a two-dimensional panel in a supersonic flow. We shall later use the following notation: a_∞ is the speed of sound in an unperturbed stream, c is the panel chord length; $d = D/\rho c^3 a_\infty$ is the bending stiffness parameter, $D = Eh^3/[12(1-\nu^2)]$ is the bending stiffness, ρ is the unperturbed air density, E is Young's modulus, $f = F/\rho c a_\infty^2$ is the stress parameter, F is the external force along the chord referred to unit panel width (positive for tension, negative for compression), M is the free stream Mach number, h is the panel thickness, m is the mass of unit panel area, $t = Ta_\infty/c$ is dimensionless time, T is time, U is the supersonic air flow velocity, X is the

Kiev. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 6, pp. 93-99, November-December, 1988. Original article submitted September 8, 1987.

distance along the panel chord, $x = X/c$ is the dimensionless distance, $Z(X, T)$ is the panel displacement, $\mu = m/\rho c$ is the ratio of the panel mass to the air mass, and ν is the Poisson ratio.

Motion of a panel in a supersonic flow and subjected to the action of a load F applied along the freely supported panel edges is described by the boundary value problem in dimensionless form

$$d\frac{\partial^4 z}{\partial x^4} + \mu\frac{\partial^2 z}{\partial t^2} - f\frac{\partial^2 z}{\partial x^2} + M\frac{\partial z}{\partial x} + \frac{\partial z}{\partial t} = 0; \tag{1.1}$$

$$z(0, t) = z(1, t) = 0, \quad \frac{\partial^2 z}{\partial x^2}(0, t) = \frac{\partial^2 z}{\partial x^2}(1, t) = 0; \tag{1.2}$$

$$z(x, t)\big|_{t=t_0} = u_0(x), \quad \frac{\partial z(x, t)}{\partial t}\Big|_{t=t_0} = u_1(t), \tag{1.3}$$

where $z = Z/c$; $x \in [0, 1]$; $t \in K \subset I \equiv [t_0, +\infty]$; K is a finite time interval, $t_0 \geq 0$, $u_0(x)$, $u_1(x)$ are the initial panel displacement and velocity distributions satisfying, by assumption, conditions assuring a single-valued solution of the boundary value problem (1.1)-(1.3) in the class of continuous functions in x, t having continuous derivatives with respect to x, t of the requisite orders [25]. The coefficients d, μ, M are positive; the coefficient f can be positive or negative.

The problem is to investigate the technical stability of the motion of a two-dimensional panel in a supersonic stream as described by the boundary value problem (1.1)-(1.3). Let us examine the Lyapunov functional

$$V[z, t] = \int_0^1 dx\left[\mu\left(\frac{\partial z}{\partial t}\right)^2 + f\left(\frac{\partial z}{\partial x}\right)^2 + d\left(\frac{\partial^2 z}{\partial x^2}\right)^2 + M\left(z^2 + 2\mu z\frac{\partial z}{\partial t}\right)\right]. \tag{1.4}$$

We use the inequality [4, 10, 12-16]

$$\int_0^1 dx\left(\frac{d^2\Phi}{dx^2}\right)^2 \geqslant \pi^2\int_0^1 dx\left(\frac{d\Phi}{dx}\right)^2 \geqslant \pi^4\int_0^1 dx\,(\Phi)^2, \tag{1.5}$$

where the continuously differentiable function $\Phi(x)$ satisfies the condition $\Phi(0) = \Phi(1) = 0$. For $f + \pi^2 d \geq 0$ the functional V has the lower bound

$$V[z, t] \geqslant \int_0^1 \left[z^2(M + \pi^2 f + \pi^4 d) + 2M\mu z\frac{\partial z}{\partial t} + \mu\left(\frac{\partial z}{\partial t}\right)^2\right]dx. \tag{1.6}$$

The quadratic form in z, $\partial z/\partial t$ in (1.6) is positive-definite if the following conditions are satisfied

$$M + \pi^2 f + \pi^4 d > 0, \ \mu(M + \pi^2 f + \pi^4 d) > M^2\mu^2. \tag{1.7}$$

The first inequality in (1.7) follows from the second. Consequently, the positive-definiteness condition for (1.4) will be

$$\mu(M + \pi^2 f + \pi^4 d) - M^2\mu^2 > 0. \tag{1.8}$$

Let us find the domain of values for which (1.8) holds for the trinomial $\mathcal{F}(M) = -M^2\mu^2 + M\mu + \mu(\pi^2 f + \pi^4 d)$. Let $r = \pi^2 f + \pi^4 d$. We consider the equation

$$-M^2\mu + M + r = 0, \tag{1.9}$$

whose roots equal

$$M_{1,2} = (2\mu)^{-1}\left(1 \pm \sqrt{1 + 4\mu^2(\pi^2 f + \pi^4 d)}\right). \tag{1.10}$$

Therefore, for

$$M_2 < M < M_1 \tag{1.11}$$

the inequality (1.8) exists, i.e., the functional V[z, t] is positive-definite when (1.11) is satisfied. If we limit ourselves to the values $M_1 > M \geq 0$, then the upper bound for M^2 will be

$$M^2 < Q \equiv \frac{f+\pi^2 d}{\mu}\left(\pi^2\mu + \frac{1+\sqrt{1+4\mu^2(\pi^2 f+\pi^4 d)}}{2\mu(f+\pi^2 d)}\right). \tag{1.12}$$

Let the second factor in (1.12) be denoted by R_1. If $R_1 \geq 1$ then (1.12) includes an analogous condition for M^2 entering into the system of sufficient Lyapunov stability condition obtained in [12-16]. We find from (1.10)

$$f > -[1/(4\mu^2\pi^2) + \pi^2 d], \tag{1.13}$$

from which it follows that f can take on negative values but not smaller than $-[1/(4\mu^2\pi^2) + \pi^2 d]$ and this means it differs from the similar condition in [4, 8, 12-15] by the first component.

Let us introduce the small positive parameter $\varepsilon \in (0, 1)$. We consider the boundaries of the greatest achievable values of the displacement and the other quantities characterizing the dynamical behavior of the system under consideration to be bounded functions of the time and the small parameter ε. Let L > 0 denote the greatest possible value of the quantity on the right in the inequality (1.12): $L = \max_{f,d}\{Q\}$ The finite time interval K being investigated will be given by using the small parameter ε: $K = [t_0, L\varepsilon^{-1}]$. We shall understand the technical stability of this system to be the stability property according to the definitions in [22-24].

<u>Definition 1.</u> The dynamic system described by the boundary value problem (1.1)-(1.3) is called technically stable in the finite time interval $K \subset I$ if the condition

$$V[z(x, t), t] \leqslant P(t), \ t \in K, \tag{1.14}$$

is satisfied along the perturbed motion z(x, t) of the boundary value problem (1.1)-(1.3) for the positive-definite functional V[z, t] if only $V[z(x, t_0), t_0] \leq a$ where the previously selected constant a = const > 0 and the previously given bounded function P(t) satisfy the inequality

$$P(t_0) \geqslant a, \ t_0 \in K. \tag{1.15}$$

in the previously mentioned time interval $K \subset I$.

<u>Definition 2.</u> If the conditions of Definition 1 are satisfied for any $K \subseteq I$, then the dynamic system characterizing the boundary value problem (1.1)-(1.3) is called technically stable in the infinite time interval I. Moreover, if $\lim_{t\to+\infty} V[z(x, t), t] = 0$ along the solution z(x, t) of problem (1.1)-(1.3), then the dynamic system is asymptotically technically stable.

Let ρ(z; ξ) denote the quantity in the right in (1.6), where ξ = (M, f, d, μ). We call ρ(z; ξ) the measure of the dynamic process (1.1)-(1.3). It is easy to see that the definitions of the TS of the original system relative to the measure ρ(z; ξ) follow from Definitions 1 and 2.

2. SUFFICIENT TS CONDITIONS IN THE PANEL FLUTTER PROBLEM ON THE BASIS OF THE METHOD OF COMPARISON

Sufficient TS conditions are obtained here in finite and infinite time intervals as well as the asymptotic TS by using the scalar Cauchy problem of comparison whose solutions depend substantially on the parameters of the dynamical process (1.1)-(1.3).

Thus, we construct the appropriate scalar comparison equation. We evaluate the derivative dV/dt along the solution z(x, t) of the problem (1.1)-(1.3).

$$\frac{dV[z(x, t), t]}{dt} = \int_0^1 dx \left\{ -2\left(\frac{\partial z(x, t)}{\partial t}\right)^2 - \right. \tag{2.1}$$

$$\left. -2M\frac{\partial z(x, t)}{\partial t}\frac{\partial z(x, t)}{\partial x} + M\left[2\mu\left(\frac{\partial z(x, t)}{\partial t}\right)^2 - 2f\left(\frac{\partial z(x, t)}{\partial x}\right)^2 - 2d\left(\frac{\partial^2 z(x, t)}{\partial x^2}\right)^2\right]\right\}.$$

Using the property of monotoneity of the product for any natural quantities, we obtain the inequality

$$\frac{1}{(\varepsilon + t)^2} V[z(x, t), t] \geqslant \frac{1}{(\varepsilon + t)^2} \rho(z(x, t); \xi) \tag{2.2}$$

along the solution of the problem (1.1)-(1.3) from (1.6). The right side in (2.1) is a regular function in t obtained after the integration with respect to the variable $x \in [0, 1]$ along the solution of the original boundary value problem (1.1)-(1.3). Moreover, this function does not exceed

$$N(t; \xi) = 2\int_0^1 dx\left[(M\mu - 1)\left(\frac{\partial z(x, t)}{\partial t}\right)^2 - M\frac{\partial z(x, t)}{\partial t}\frac{\partial z(x, t)}{\partial x} - M(f + \pi^2 d)\left(\frac{\partial z(x, t)}{\partial x}\right)^2\right]. \tag{2.3}$$

The integrand of the function $N(t; \xi)$ is a sign-indefinite quadratic form in $\partial z/\partial t$, $\partial z/\partial x$. By using (2.2) and (2.3), we form the function

$$\tilde{\Phi}(t; \xi) = N(t; \xi) - \frac{1}{(\varepsilon + t)^2} \rho(z(x, t); \xi). \tag{2.4}$$

The inequality $N(t; \xi) - \frac{1}{(\varepsilon + t)^2} V[z(x, t), t] \leqslant \tilde{\Phi}(t; \xi)$ holds. There is a majorant

$$\tilde{\Phi}(t; \xi) \leqslant N(t; \xi) - \frac{\mu}{(\varepsilon + t)^2}\int_0^1 \left(\frac{\partial z(x, t)}{\partial t}\right)^2 dx. \tag{2.5}$$

for the function $\tilde{\Phi}(t; \xi)$. We let $\Phi(t; \xi)$ denote the right side in (2.5) along the solution of the problem (1.1)-(1.3). Let us note that the quantities in (2.2)-(2.5) depend on M, f, d, μ as on parameters satisfying conditions (1.7)-(1.13). Along the solution of the boundary value problem (1.1)-(1.3)

$$dV[z(x, t), t]/dt \leqslant (\varepsilon + t)^{-2} V[z(x, t), t] + \Phi(t; \xi). \tag{2.6}$$

Let us examine the function $\sigma(t; \xi) = \int_0^1 \Phi(\tau; \xi)\, d\tau$, along the solution of the problem (1.1)-(1.3) by the condition of a problem continuous in a given interval K. We assume $k(t) = V[z(x, t), t] - \sigma(t; \xi)$. Then we write the inequality (2.6) in the form

$$dk(t)/dt \leqslant (\varepsilon + t)^{-2}[k(t) + \sigma(t; \xi)]. \tag{2.7}$$

The function $\Phi(t; \xi)$ contains the time explicitly in its coefficients. Its integrand is sign-definite here for not all values of $t \in K$. It is easy to see that only under the conditions

$$t > \Theta \geqslant t_0, \quad \Theta \equiv \sqrt{\frac{2\mu(f + \pi^2 d)}{M + \mu - 4(f + \pi^2 d)}} - \varepsilon, \tag{2.8}$$
$$\max\{0; 4(f + \pi^2 d) - \mu\} < M < M_1$$

is the integrand in $\Phi(t; \xi)$ negative definite as a quadratic form in $\partial z/\partial t$, $\partial z/\partial x$. Therefore, the function $\sigma(t; \xi)$ is also a sign-definite bounded function in the domain K only under the conditions (2.8). Starting from the inequality (2.7), we consider the comparison differential equation

$$dy/dt = (\varepsilon + t)^{-2}[y + \sigma(t;\xi)],\ t \in K \quad (2.9)$$

under the initial conditions

$$y(t_0) = y_0 \geqslant V[z(t_0), t_0], \quad (2.10)$$

where

$$V[z(t_0), t_0] = \int_0^1 dx \left[\mu u_1^2(x) + f\left(\frac{\partial u_0(x)}{\partial x}\right)^2 + d\left(\frac{\partial^2 u_0(x)}{\partial x^2}\right)^2 + \mathrm{M}\left(u_0^2(x) + 2\mu u_0(x) u_1(x)\right)\right]. \quad (2.11)$$

The general solution of the Cauchy problem (2.9), (2.10) is written as

$$y(t, t_0) = y_0 \exp\left(\frac{1}{\varepsilon + t_0} - \frac{1}{\varepsilon + t}\right) + \exp\left(-\frac{1}{\varepsilon + t}\right) \int_{t_0}^{t} \exp\left(\frac{1}{\varepsilon + \tau}\right) \Phi(\tau;\xi)\, d\tau - \sigma(t;\xi).$$

Taking into account that $V[z(t_0), t_0] \equiv k(t_0)$, we obtain the estimate

$$V[z(x, t), t] \leqslant P(t;\xi,\varepsilon) \equiv y_0 \exp\left(\frac{1}{\varepsilon + t_0} - \frac{1}{\varepsilon + t}\right) + \exp\left(-\frac{1}{\varepsilon + t}\right)\int_{t_0}^{t} \exp\left(\frac{1}{\tau + \varepsilon}\right)\Phi(\tau;\varepsilon)\, d\tau \quad \forall t \in K. \quad (2.12)$$

from (2.7) and (2.10) by means of the theorem on differential inequalities [22-24, 26] for the Lyapunov functional (1.4) along the solution of the problem (1.1)-(1.3). Taking account of the continuity property of the function $P(t;\xi,\varepsilon)$ in a given time interval K for arbitrary $\varepsilon \in (0, 1)$ and the above-mentioned parameters ξ, the TS of the system under consideration in a finite time interval follows from (2.12).

Indeed, let the constant C = const > 0 be previously known such that the following estimate holds

$$|\sigma(t;\xi)| \leqslant C \quad \forall t \in K \quad (2.13)$$

for the above-mentioned conditions on the parameter ξ. Then by using integration by parts and majorizing, we find the inequality

$$P(t;\xi,\varepsilon) \leqslant \exp\left(-\frac{1}{\varepsilon + t}\right)\exp\left(\frac{1}{\varepsilon + t_0}\right)(y_0 + C). \quad (2.14)$$

The function on the right in (2.14) does not exceed

$$\exp\left(-\frac{1}{\varepsilon + L\varepsilon^{-1}}\right)\exp\left(\frac{1}{\varepsilon + t_0}\right)(y_0 + C). \quad (2.15)$$

during the interval K. Hence, taking account of condition (2.10) the technical stability of the motion of a two-dimensional panel in a finite time interval K that is in a supersonic flow and is subjected to the action of a strong load along its free edges follows.

If the function $\Phi(t;\xi)$ is such that the integral $\int_{t_0}^{t} \exp\left(\frac{1}{\varepsilon + \tau}\right)\Phi(\tau;\xi)\, d\tau$ is a continuous bounded function in any time interval $K \subseteq I$ and has growth in each $K \subseteq I$ not more rapid than the function $\exp\left(\frac{1}{\varepsilon + t}\right)$, then as follows from (2.12), the original system (1.1)-(1.3) defined in each interval $K \subseteq I$ is technically stable in an infinite time interval, i.e., in this case a previously assigned continuous bounded function B(t) found in each $K \subseteq I$ can be indicated such that the estimate

$$\int_{t_0}^{t} \exp\left(\frac{1}{\varepsilon + \tau}\right)\Phi(\tau;\xi)\, d\tau \leqslant B(t)\exp\left(\frac{1}{\varepsilon + t}\right), \quad t \in K \subseteq I \quad (2.16)$$

will be satisfied under the condition $B(t)+y_0\exp\left(\frac{1}{\varepsilon+t_0}\right)\exp\left(-\frac{1}{\varepsilon+t}\right)\geqslant 0,\ t\in K\subseteq I$. In particular, condition (2.16) can be of the form $\exp\left(\frac{1}{\varepsilon+t}\right)\geqslant\int\limits_{t}^{t}\exp\left(\frac{1}{\varepsilon+t}\right)\Phi(\tau;\xi)\,d\tau$ for all K $\subseteq$ I.

In this case, as t → +∞ we have $P(t;\xi,\varepsilon)\leqslant y_0\exp\left(\frac{1}{\varepsilon+t_0}\right)+1$.

Utilizing the condition (2.13) for all K ⊆ I, we obtain the estimate

$$P(t;\xi,\varepsilon)\leqslant\exp\left(\frac{1}{\varepsilon+t_0}\right)(y_0+C)\quad\forall t\in I. \tag{2.17}$$

from the inequality (2.14). Hence, taking account of (2.10) and (2.12), the TS of panel motion in a supersonic flow in an infinite time interval follows under the condition (2.13) for all K ⊆ I. As is seen from (2.15), the interval K can here be determined by using the small parameter ε → 0. Then from (2.15) for ε → 0 we find $\exp\left(-\frac{\varepsilon}{\varepsilon+L}\right)\exp\left(\frac{1}{\varepsilon+t_0}\right)(y_0+C)\to\exp\left(\frac{1}{t_0}\right)(y_0+C)$, $t_0\neq 0$. Since ε ≠ 0 in this case, then in place of (2.17) we have the strict inequality $P(t;\xi,\varepsilon)<\exp\left(\frac{1}{t_0}\right)(y_0+C)$.

If the original system (1.1)-(1.3) is technically stable in I and, moreover, the condition

$$P(t;\xi,\varepsilon)\to 0 \text{ for } t\to+\infty \tag{2.18}$$

is satisfied for given ξ, ε, then the system (1.1)-(1.3) is technically asymptotically stable. In particular, condition (2.18) will be satisfied if the condition

$$\int\limits_{t_0}^{t}\exp\left(\frac{1}{\varepsilon+\tau}\right)\Phi(\tau;\xi)\,d\tau\to-y_0\exp\left(\frac{1}{\varepsilon+t_0}\right).$$

is valid.

There results from (1.12) that satisfaction of the inequality $M^2\geq Q$ implies spoilage of the positive-definiteness of the functional (1.4). However, this is inadequate for speaking about the TS of the original system. The original process (1.1)-(1.3) will evidently be technically unstable for given parameters if the inequality $V[z(x,t),t]>P(t;\xi,\varepsilon)$ holds for at least one time t in the considered (finite or infinite) time interval. As follows from the estimate (2.12), one of the technical instability conditions of the process (1.1)-(1.3) is the condition $\int\limits_{t_0}^{t}\exp\left(\frac{1}{\tau+\varepsilon}\right)\times\Phi(\tau,\xi)\,d\tau\to+\infty$ for t ∈ K or t ∈ I, respectively. We here take $M_k=M_1$ as the critical Mach number, where M_1 is given according to (1.10).

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